DR. BABASAHEB AMBEDKAR MARATHWADA UNIVERSITY, AURANGABAD.

Curriculum under Choice Based Credit & Grading System

M.Sc. I Year

Mathematics

Semester–I to II

run at college level from the

Academic Year 2015-16 & onwards

DR. BABASAHEB AMBEDKAR MARATHWADA UNIVERSITY, AURANGABAD

DEPARTMENT OF MATHEMATICS

Syllabus for M.A. / M. Sc. (Mathematics) Semester I, II, III, and IV
Under Academic Flexibility of the Department
Credit Based Grading System
W.E.F. JUNE – 2011
And modified in June 2014.

The M. A. / M. Sc. (Mathematics) course consists of four semesters.

In Semesters I and II a student has to study four **core** Courses and one **Elective** course. In Semesters III and IV he/she has to study two **core** Courses, three **elective** courses and at least one **service course** from any other department. The students of other Departments **may opt the course MAT-503** offered for semester III as a **service course**. Unit wise distribution of the syllabus for the courses currently taught is given.

The M. A. / M. Sc. (Mathematics) course will be of 120 credits. The credits obtained from other Department will be appropriately converted.

SEMESTER- I (Core Courses)

MAT401 - Advanced Abstract Algebra -I

MAT402 - Real Analysis -I

MAT403 - Topology -I

MAT404 - Complex Analysis -I

Elective Courses (Any One)

MAT421 – Differential Equations -I.

MAT422 - Advanced Discrete Mathematics -I.

SEMESTER –II (Core Courses)

MAT411 - Advanced Abstract Algebra -II

MAT412 - Real Analysis -II

MAT413 - Topology -II

MAT414 - Complex Analysis -II

Elective Course (Any one of the following)

MAT431	-	Differential Equations -II
MAT432	-	Advanced Discrete Mathematics -II.

SEMESTER III (Core Courses)

MAT501	-	Functional Analysis
MAT502	-	Partial Differential Equations

Elective Courses (Any three of the following)

MAT521	-	Programming in C
MAT522	-	Fluid Mechanics -I
MAT524	-	Numerical Analysis .
MAT525	-	Lattice Theory
MAT526	-	Operations Research -I.
MAT527	-	Reaction diffusion theory - I
MAT528	-	Difference Equations -I

SEMESTER IV (Core Courses)

MAT511	-	Linear Integral Equations
MAT512	-	Mechanics

Elective Courses (Any three of the following)

MAT531	-	MATLAB Programming
MAT532	-	Fluid Mechanics -II
MAT534	-	Fuzzy Mathematics
MAT535	-	Linear Algebra
MAT536	-	Operations Research -II.
MAT537	-	Reaction diffusion theory - II
MAT538	-	Difference Equations -II

Course No: MAT401 **Number of Credits: 6**

Semester –I Advanced Abstract Algebra- I

Unit- I

Binary relation, binary operation, function,group,subgroup and their properties. Order of a group, element and a subgroup Generator, cyclic group,Lagranges theorem, Fermats and Eulers theorem and their consequences. (15 lectures)

Unit- II

Normal subgroup, quotient group and their properties and axamples. Homomorphism, kernel, image of a homomorphism. Isomorphism and related theorems, Fundamental theorem of group homomorphism, automorphism, conjugacy and G-sets. (15 lectures)

Unit- III

Permutation groups and related concepts and results. Center, normalizer,commutator of a group, derved group, Cayles theorem. (15 lectures)

Unit – IV

Normal series, solvable and nilpotent group and their properties, direct products, simplicity of alternating group. (15 lectures)

Unit- V

Fundamental theorem of finitely generated abelian group, invariants of finite abelite abelian group, Sylow theorems and applications. (15 lectures)

Text Book:

Basic Abstract Algebra, by P. B. Bhattacharya, S. K. Jain and S. R. NagPaul Cambridge (Indian Edition)Chapter Number:4,5,6,7,8 related topics.

Reference Books:

1. Topics in algebra, I. N. Herstein: Wiley (Indian Edition)
2. Contemporary Abstract Algebra by J.A. Gallian, Narosa.

Course No :MAT402 **Number of Credits :6**

Semester – I Real Analysis- I

Unit – I
Definition and existence of Riemann-Stieltjes integral, Properties of the integral, Integration and Differentiation, The fundamental theorem of calculus, Examples. (15 lectures)

Unit – II
Integration of vector valued functions. Rectifiable curve. Examples. Sequences and series of functions. Point wise and uniform convergence. Cauehy criterion for uniform convergence. Weierstrass M-test, uniform convergence and continuity, uniform convergence and Riemann-Stielljes integration. Examples. (15 lectures)

Unit – III
Uniform convergence and Differential, The Stone – Weierstrass theorem, Examples. Power series, Abel's and Taylor's theorems, Uniqueness theorem for power series. Examples. (15 lectures)

Unit – IV
Functions of several variables, Linear transformations, Derivatives in an open subset of R^n, Chain rule, Examples (15 lectures)

Unit – V
Partial derivations. Interchange of the order of differentiation, The inverse function theorem, The implicit function theorem Jacobins, Derivatives of higher order, Differentiation of integrals. Examples, (15 lecturer)

Text Book:
Walter Rudin, Principles of Mathematical Analysis, (3rd Edition) McGraw Hill, Kogakusha 1976.

Articles:
6.1 to 6.27, 7.1 to 7.18, 7.26, 7.27, 8.1 to 8.5, 9.1 to 9.21, 9.24 to 9.29, 9.38 to 9.42

Reference Books:

1. T. M. Apostol, mathematical Analysis, Narosa, New Delhi, 1985.

2. J. C. Burkill and H. Burkill, A second course in Mathematical Analysis, Cambridge University Press, 1970.

3. S. L. Lang, Analysis- I and II, Addison Wesley, 1969.

Course No :MAT403 **Number of Credits :6**

Semester - I Topology - I

Unit – I

Prerequisites: Partially ordered sets, Maximal and minimal elements, cardinality, special cardinals countable and uncountable sets, Axiom of choice continuum hypothesis, principle of inductions metric spaces, definition and Examples, continuous map, open sets properties of open sets, characterizations of continuity. (15 lectures)

Unit – II

Definition and examples of topological spaces, closed sets, closure of a set, properties of closure of sets, interior of a set and their properties, frontier of sets and its relation ship with closure and interior of sets neighbourhood of a point, Neighbourhood system, accumulation point and derived set. (15 lectures)

Unit – III

Bases and sub bases and related theorems new spaces from old. Sub spaces, continuous functions product spaces, weak topologies and related theorems open closed maps projection maps. (15 lectures)

Unit – IV

Evaluation map and related results Quotient spaces sequences in a topological space, Inadequacy of sequences, first countable spaces. (15 lectures)

Unit – V

Directed sets, nets, convergence of nets, cluster point, subnet, ultra net, filter, convergence of filters, ultra filters, fixed and free filters, results on these concepts.

(15 lectures)

Textbook: Stephan Willard: General Topology, Addison Wesley (1970)

Chapter 1 (Sec. 1.8 to 1.21 and Sec, 2.1 to 2.8)
Chapter 2 (Complete),
Chapter 3 (up to sec. 9.3)
Chapter 4 (Complete)

Reference Books:

1. Steen & J. Seecatch: Counter examples in Topology, Holt, Rinehart and instant, N, Y. (1970).
2. W. J. Pervin: Foundation of general Topology Academic press N.Y.
3. S. T. Hu. : Elements of general Topology, Holden.
4. James Munkres: Topology, A first course, Prentice Hall of India Pvt. Ltd.

Course No :MAT404 **Number of Credits :6**

Semester – I - Complex Analysis - I

Unit- I

The Complex number system:

The field of complex numbers, The complex plane, Rectangular and polar representation of complex numbers; Intrinsic function on the complex field; The Complex plane. (15 lectures)

Unit - II

Metric spaces and Topology of C:

Definition and examples of metric spaces; connectedness; sequence and completeness; compactness; continuity; Uniform convergence. (15 lecturer)

Unit- III

Elementary properties and examples of Analytic functions:

Power series; The exponential function; Trigonometric and hyperbolic functions; Argument of nonzero complex number; Roots of unity; Branch of logarithm function. Analytic functions; cauchy Riemann Equations; Harmonic function; (15 lectures)

Unit - IV

Analytic functions as a mapping; Mobius transformations; linear transformations; The point at infinity; Bilinear transformations,

Complex Integration: power series representation of analytic functions; zeros of an analytic function. (15 lectures)

Unit – V

The index of a closed curve; cauchy's theorem and integral formula; Gaursat's Theorem; Singularities: Classification of singularities; Residues; The argument principle. (15 lectures)

Text Books:

1. John B. Conway; Functions of one complex variable, Narosa Publishing House, 2002.
2. J. V. Deshpande; Complex Analysis, Tata McGraw- Hill Publishing Company Limited, 1989.

Unit-I: Chapter-I: § 2,3,4 in [1] & 1.3 & 1.4 in [2]

Unit-II: Chapter – II: §1,2,3,4,5,6 in [1]

Unit – III: Chapter –VI: § 6.1,6.2,6.3,6.4,6.5,6.6 in [2] & Chapter - VII: § 7.1,7.2,7.3, in [2]

Unit- IV: Chapter- III: § 3 in [1] & Chapter – IV: § 2, 3 & 4 in [1] &
Chapter – 2: § 2.1,2.2,2.3, in [2]

Unit – V: Chapter – IV: § 5 & 8 in [1] & Chapter V: § 1,2 & 3 in [1].

References:

1. Herb Silverman; Complex Variables, Haughton Mifflin Company Boston, 1975.
2. Ruel V. Churchill; Complex variables and applications, McGraw – Hill Publishing Company 1990.

Course No :MAT421 **Number of Credits :6**

Semester – I -Differential Equations - I

Unit – I

Existence, uniqueness and Continuation of solutions: Introduction, Method of successive approximations for the initial value problem $y^1=f(x,y)$, $y(x_0)=y_0,$ The Lipschitz condition. Notation and Definitions, Peano's existence theorem, maximal and minimal solutions, continuation of solutions. (15 lectures)

Unit – II

Existence theorems for system of differential equations: Picard-Lindelof theorem, Peano's existence theorem, Dini's derivatives, differential inequalities. (15 lectures)

Unit – III

Integral Inequalities: Gronwall- Reid-Bellman inequality and its generalization, Applications: Zieburs theorem, Peron's criterion, Kamke's uniqueness theorem. (15 lectures)

Unit – IV

Linear systems: Introduction, superposition principle, preliminaries and Basic results, Properties of linear homogeneous system, Theorems on existence of a fundamental system of solutions of first order linear homogeneous system, Abel-Liouville formula. (15 lectures)

Unit – V

Adjoint system, Periodic linear system, Floquet's theorem and its consequences, Applications, Inhomogeneous linear systems, applications. (15 lectures)

Text Book:

1. E. A. Coddington: An Introduction to Ordinary Differential Equations. Prentice-Hall international, Inc. Englewood Cliffs (1961).

   Chapter 6: Article 4&5.

2. Shair Ahmad and M. Rama Mohana Rao: Theory of Ordinary Differential Equations with Applications in Biology and Engineering, Affiliated East-West Press (1999)
   Chapter – 1: Article 1.1 to 1.5
   Chapter – 2: Article 2.1 to 2.3

References:

1. P. Hartman: Ordinary differential Equations, 2^{nd} edition, SIAM, (2002.)
2. W. T. Reid: Ordinary Differential Equations, John Wiley, New York, (1971).
3. E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations, McGraw-Hill, New York, (1955)

Course No :MAT422 **Number of Credits :6**

Semester – I - ADVANCED DISCRETE MATHEMATICS - I

Unit – I

Formal Logic: Statements, symbolic representation, tautologies. Semi groups and monoids: Definitions and examples of semi groups and Monoids
(15 Lecture)

Unit- II

Homomorphism of semigroups and monoids, congruence relation and quotient semigroups, Sub semigroups and submonoids, direct products, basic homomorphism theorem. (15 Lecture)

Unit- III

Lattices: Lattices as partially ordered sets, their properties, lattices as algebraic systems, sub lattices, direct products and homomorphism, some special lattices eg complete, complemented and distributive lattices.
(15 Lecture)

Unit- IV

Boolean algebras: Boolean algebras as lattices, various Boolean identities, the switching algebra example, sub algebra, direct product and homomorphism, join-irreducible elements (15 Lecture)

Unit- V

Atoms and midterms, Boolean forms and their equivalence, midterm Boolean forms, (excluding free Boolean algebras), sum and products of canonical forms. Minimization of Boolean functions, applications of Boolean algebra to switching theory (using AND, ok and NOT gates), the Karnaugh Map method.
(15 Lecture)

Text Book:

1. **J. P. Tremblay and R. Manohar**: Discrete Mathematical structures with Applications to Computer science, McGraw-Hill Book Co., 1997.

Chapter 1 (Sections 1.1 to 1.3), Chapter 3 (Sections 3.1 and 3.2), Chapter 4 (Sections 4.1 to 4.4)

Reference Books:

1. **Seymour Lepschutz**: Finite Mathematics, McGraw-Hill, New York.
2. **S. Wiitala**: Discrete Mathematics - A Unified Approach, McGraw-Hill.
3. **J. E. Hhopcroft and J.D. Ullman**: Introduction to Automata Theory, Languages and Computation, Narosa, New Delhi.
4. **C. L. Liu**: Elements of discrete Mathematics, McGraw-Hill Book Co.

Course No :MAT411 **Number of Credits :6**

Semester – II - Advanced Abstract Algebra -II

Unit- I
Preliminaries of rings, definition, types, subring, ideal, prime, maximal ideas, nil, nilpotent ideals and their properties. Quotient ring, Homomorphism, isomorphism and related results. UFD, PID, Euclidean domain, polynomial rings and their properties.

Chapter 3 from[1], (15 lectures)

Unit – II
Vector spaces, subspaces, generating set, linear dependence and indepeedence, basis and dimension, quotient space, homomorphism, dual space, inner product space and modules.

Chapter 4 from[1], (15 lectures)

Unit - III
Liner transformation and their properties, characteristic roots, triangular canonical form.

Chapter 6 (upto Theorem 6.4.2) from [1,] (15 lectures)

Unit – IV
Extension fields, irreducible polynomials, algebraic extension and their properties, splitting field, normal extension, multiple roots, finite fields, separable extension.

Chapter 15 and 16 from [2,] (15 lectures)

Unit – V
Automorphism groups, fixed field, fundamental theorm of Galois theory, polynomials solvable by radicals, ruler and compass constructions. Related topics from chapter 17,18 from [1] (15 lectures)

Text Book:

1) Topics in Algebra by I. N. Herstein, Wiley.

2) Basic Abstract Algebra by Bhattacharya, Jain and NagPaul, Cambridge (Indian Edition)

Reference book: Contemporary Abstract Algebra by J. A. Gallian,Narosa.

Course No :MAT412 **Number of Credits :6**

Semester – II -Real Analysis -II

Unit – I

Measure on the real line. Lebegue outer measure, measurable sets. Regularity. Measurable functions. Borel and lebegue measurability. Examples. (15 lectures)

Unit – II

Integration of functions of a Real variable. Integration of a simple function. Integration of non-negative functions. The general integral. Integration of series. Examples.
(15 lectures)

Unit – III

Riemann and Lebeque Integrals, Differentiation. The four derivates, Functions of bounded variations. Lebegue's differentiation theorem differentiation and Integration. Examples. (15 lecturer)

Unit – IV

Abstract Measure spaces. Measures and outer measures Extension of a measure. Uniqueness of the extension. Completion of a measure spaces. Integration with respective to a measure. Examples. (15 lecturer)

Unit – V

The L^p spaces. Convex functions. Jensen's inequality. The inequalities of Holder and Minkowski Completeness of L^p (μ) Convergence in measure. Almost uniform convergence. Examples. (15 lecturer)

Text Book:

G. de Barra, Measure Theory and Integration. Wiley Eastern Ltd. 1981. Reprint 2003.

Articles: 2.1-2.5, 3.1 – 3.4, 4.1, 4.3 - 4.5, 5.1 – 5.6, 6.1 – 6.5, 7.1 and 7.2

Reference Books:

1. P. K. Jain and P. V. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd. Publication New Delhi. 1986 (Reprint 2000)
2. P. R. Halmos, Measure Theory, Von No strand, Princeton 1950
3. R. G. Bartle, The elements of Integration, John Wiley, New York 1966.
4. I. K Rana, An Introduction to measure and Integration, Narosa, Delhi 1997.

Course No :MAT413 **Number of Credits :6**

Semester – II -Topology -II

Unit – I
Separation axioms, $T_{0,}$ T1, $T_{2,}$ space their properties and characterizations, regular spaces, T_3 spaces, characterizations, Hereditariness of these concepts, completely regular and tychonoff spaces and their characterizations.(15 lectures)

Unit – II
Normal spaces and T_4 spaces, urysohrns lemma, tietze theorem on normal spaces (without proof) cover, point finite cover, shrinkable cover of topological spaces and their properties, accountability properties, second countable spaces, lindelof spaces and their properties. (15 lectures)

Unit – III
Compactness. Definition and examples, characterization of compactness, sequentially and countably compact spaces, locally compact specs and their properties, compactification, one point compectification, Stone-cech compactification. (15 lectures)

Unit – IV
Para compactness, local finiteness, Metrizable spaces, lebesgue covering lemma, Urysohns motrization theorem, metrizebility of T_0 spaces, (15 lectures)

Unit – V
Connected spaces, mutually separated sets, characterizations and properties of connected spaces, components, simple chain, path wise and local connectedness. (15 lectures)

Text Book: Stephan Willard: General Topology Addison Wesley publication Co. (1970)

Chapter 5 (Complete) chapter- 6 (Section17.1 to 19.5, 20.1 to 20. 10)
Chapter 7 (Section 22.1 to 23.2, 23.5), Chapter 8 (Section 26.1 to 27.13)

Reference Books:

1. Steen & J. Seecatch: Counter examples in Topology, Holt, Rinehart and instant, N, Y. (1970).
2. W. J. Pervin: Foundation of general Topology Academic press N.Y.
3. S. T. Hu. : Elements of general Topology, Holden.
4. James Munkres: Topology, A first course, Prentice Hall of India Pvt. Ltd.

Course No :MAT414 **Number of Credits :6**

Semester - II -Complex Analysis -II

Unit – I

Compactness and convergence in the space of Analytic functions:

Spaces of analytic functions; The weierstrass factorization theorem; factorization of the sine function; The gamma function; The Riemann zeta function.

(15 lectures)

Unit – II

Harmonic functions:

Basic properties of Harmonic functions and comparison with analytic function; Harmonic functions on a dick; Poisson integral formula; positive harmonic functions.

(15 lectures)

Unit – III

Entire functions; Jensen's formula; The Poisson-Jenson formula; The genus and order of an entire function. Hadamard factorization Theorem; (15 lectures)

Unit – IV

Univalent functions; the class S; the class T; Bieberbach conjucture; sub class of s; (15 lectures)

Unit – V

Analytic continuation: Basic concepts; special functions. (15 lectures)

Text Books:

1. John B. Conway; Functions of one complex variable, Narosa Publishing House, 1980.
2. Herb Silverman; Complex Variables Houghton Mifflin Company Boston 1975.

Unit – I	:	Chapter – VI: § 2,5,6,7 & 8 in [1]
Unit – II	:	Chapter – X: § 1& 2 in [1]
		And Chapter- X: § 10.1, 10.2 & 10.3 in [2]
Unit – III	:	Chapter- XI: § 1,2 & 3 in [1]
Unit – IV	:	Chapter XII: § 12.1& 12.2 in [2]
Unit – V	:	Chapter – XIV: § 14.1 & 14.2 in [2]

Reference Books:

1. L. V. Ahlfors: Complex Analysis, McGraw-Hill International Editions, 1979.
2. Ruel V. Churchill and J. W. Wran; Complex variables and applications, McGraw-Hill publishing Company – 1990.

Course No :MAT431 **Number of Credits :6**

Semester – II -Differential Equations - II

Unit- I
Preliminaries, Basic Facts: Superposition principles, Lagrange Identity, Green's formula, variation of constants, Liouville substitution, Riccati equations Prufer Transformation. Higher order linear equations. (15 lectures)

Unit – II
Maximum Principles and their extensions, Generalized maximum principles, initial value problems, boundary value problems. (15lectures)

Unit –III
Theorems of strum; sturm's first comparison theorem, sturm's separation theorem, strums second comparison theorem. (15lectures)

Unit – IV
Sturm-Liouville boundary Value Problems: definition, eigenvalues, eigenfunctions, orthogonality. (15lectures)

Unit – V
Number of zeros, Non oscillatory equations and principal solutions, Non oscillation theorems. (15 lectures)

Text Books:

1. Philip Hartman: Ordinary differential Equations, 2nd Edition SIAM, 2002.
   Chapter – XI: Article 1 to 7. Chapter – 4 – article 8 only.
2. M. H. Protter and H. F. Weinberger, Springer: Maximum Principles in Differential Equations – Springer Verlag, New York, Inc, 1984.
   Chapter 1. Articles 1 to 4.

Reference Books:

1. W. T. Reid: ordinary differential Equations, John Wiley N.Y. (1971).
2. E. A. Coddington and N. Levinson: Theory of Ordinary differential Equation, McGraw-Hill, New York, (1955).

Course No :MAT432 **Number of Credits :6**

Semester – II -ADVANCED DISCRETE MATHEMATICS -II

Unit – I

Definiton of (undirected) graph, paths, circuits, cycles and subgraphs, degree of a vertex connectivity, planar graphs and their properties.

Unit-II

Trees, rulers formula for connected planar graphs. Complete graphs, Kuratowski's theorem (statement only) spanning trees, cutsets, fundamental cut-sets and cycles, minimal spanning trees and Kruskal's (statement only) algorithm, matrix representation of graphs,

Unit-III

Euler's theorem on the existence of Eulerian paths and circuits, directed graphs, in degree and out degree of a vertex, weighted undirected graphs, strong connectivity, directed trees, search trees,

Unit-Iv

Introductory computability theory:

Finite state machines and their transition table diagrams, equivalence of finite state machies, reduced machines, homomorphism, finite automata, acceptors, no-deterministic finite automata.

Unit-V

Grammers and languages: Phase structure grammars, rewriting rules, derivations, sentential forms, language generated by a grammar, regular, contest free and contest sensitive grammers and languages.

Text Books:

1. **J. P. Tremblay and R. Manohar**: Discrete Mathematical structures with Applications to Computer science, McGraw-Hill Book Co., 1997.

Sections 3.3, 4.6, and 5.1 to 5.6

2. **C. L. Liu**: Elements of discrete Mathematics, McGraw-Hill Book Co.

Sections 6.5 and 7.1 to 7.7

Reference Books:

1. **Seymour Lepschutz**: Finite Mathematics, McGraw-Hill, New York.
2. **S. Wiitala**: Discrete Mathematics - A Unified Approach, McGraw-Hill.
3. **J. E. Hhopcroft and J.D. Ullman**: Introduction to Automata Theory, Languages and Computation, Narosa, New Delhi.